Case Study
A Parametric Model for the Cost per Flight Hour (CPFH)

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Applicability of Cost Estimating Techniques (1/2)

Case Study: A Parametric Model for the CPFH

Applicability of Cost Estimating Techniques (2/2)

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<th>Concept</th>
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<th>Production</th>
<th>Utilization/Support</th>
<th>Retirement</th>
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<td>Parametric</td>
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<tr>
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<td>Expert survey</td>
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<tr>
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<tr>
<td>Activity based</td>
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<td>● ●</td>
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</tbody>
</table>

Applicability of cost estimating techniques over the LC stages (amount of points indicates suitability)

Source: AAP-48 draft Ed.3
Analogy

\[ Cost = f(Cost') \]

\[ O&S\ Cost \approx 2.10 \cdot \text{Procurement} \]

\[ CPFH = \frac{O&S\ Cost}{\text{FLHRs}} \]

\[ O&S\ Cost \approx 2.34 \cdot \text{Procurement} \]

Parametric

Cost = f(p₁, p₂, p₃, ..., pₙ)

Source: Mike Carey, Naval Center for Cost Analysis, 41st annual DoDCAS (2008)
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### Engineering (Build-Up)

\[
\text{Cost} = \sum_{i=1}^{n} \text{Cost}_i
\]

**O&S Cost breakdown**

1. 0 Unit-level manpower
2. 0 Unit operations
3. 0 Maintenance
4. 0 Sustaining support
5. 0 Continuing system improvements
6. 0 Indirect cost

**TOTAL**

*Source: GAO-09-3SP Cost Estimating and Assessment Guide (2009)*
Construction of a parametric model

Known systems

- Cost of System A
  - Parameter_1
  - Parameter_2
  - Parameter_3
  - Parameter_4
  - Parameter_5
  - ...

- Cost of System B
  - Parameter_1
  - Parameter_2
  - Parameter_3
  - Parameter_4
  - Parameter_5
  - ...

- Cost of System C
  - Parameter_1
  - Parameter_2
  - Parameter_3
  - Parameter_4
  - Parameter_5
  - ...

Parametric model

Cost Estimating Relationships (CERs)
Pre-Analysis considerations: Constraints & Requirements

• Use the available (small) sample of 22 systems that HAF operates
• Exclude indirect cost
• Search for cost drivers that are easily accessible and quantifiable
• The selected model must:
  ❖ not include more than two cost drivers
  ❖ be significant at the 5% level
  ❖ capture at least 75% of the CPFH variance
  ❖ have valid confidence & prediction intervals
  ❖ make sense
Hellenic Air Force fleet

Trainers
- T-41D
- T-6A II
- T-2E

Fire fighters
- CL-215
- CL-415
- PZL

Fighters
- F-16C/D
- F/RF-4E
- M2000-5
- A-7H

AEW&C
- EMB-145H

VIP
- EMB-135

Helicopters
- AB-205
- B-212
- AS-332C1
- A-109E

Transporters
- C-130H
- C-27J

Source: haf.gr
How will the model ‘perceive’ the systems, according to the above independent variables?

- Length
- Empty weight
- MTOW
- SFC (max)
- Speed (max)
- Ceiling
Independent variables
• Length
• Empty weight
• MTOW
• SFC (max)
• Speed (max)
• Ceiling

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Independent variables
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• Length
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• MTOW
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• Ceiling

How will the model ‘perceive’ the systems, according to the above independent variables?
Multicollinearity issues

Different variables contain the same information!!!
(They are highly correlated and one can be linearly predicted from the other(s))
Model selection

Simple CER

Log\(CPFH\) = \(a_0 + a_1 \cdot \text{Log}(MTOW)\)

\(R^2 = 0.69\)

Complex CER

Log\(CPFH\) = \(\beta_0 + \beta_1 \cdot \text{Log}(\text{Empty weight}) + \beta_2 \cdot \text{Log}(SFC)\)

\(R_{adj}^2 = 0.82\)

Akaike Criterion
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ANOVA table

Call: lm(formula = LogCPFH ~ LogEMPTY + LogSFC)

Residuals:
     Min       1Q   Median       3Q      Max
-0.42125 -0.08515 -0.02154  0.09199  0.50650

Coefficients:

|                     | Estimate | Std. Error | t value | Pr(>|t|) |
|---------------------|----------|------------|---------|----------|
| (Intercept)         |          |            |         | ***      |
| LogEMPTY            |          |            |         | ***      |
| LogSFC              |          |            |         | ***      |

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2553 on 19 degrees of freedom
Multiple R-squared:  0.8385,  Adjusted R-squared:  0.8215
F-statistic: 49.31 on 2 and 19 DF,  p-value: 3.009e-08

Correlation of Coefficients:

|                     | (Intercept) LogEMPTY LogSFC |
|---------------------|-----------------------------|----------------|
| LogEMPTY            | -0.99                       | 0.17 |
| LogSFC              | 0.17                        | -0.13 |
# Residuals diagnostics

<table>
<thead>
<tr>
<th>Test</th>
<th>Null hypothesis</th>
<th>p-value</th>
<th>Reject the null hypothesis at the 5% sig. level?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shapiro-Wilk normality test</td>
<td>normality</td>
<td>0.161</td>
<td>NO</td>
</tr>
<tr>
<td>Breusch-Pagan test for heteroscedasticity</td>
<td>constant variance</td>
<td>0.332</td>
<td>NO</td>
</tr>
<tr>
<td>Durbin-Watson test for autocorrelation</td>
<td>no autocorrelations</td>
<td>0.342</td>
<td>NO</td>
</tr>
<tr>
<td>Two-sided t-test with Bonferroni adjustment</td>
<td>no outliers</td>
<td>0.714</td>
<td>NO</td>
</tr>
</tbody>
</table>

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**Case Study: A Parametric Model for the CPFH**

![Distribution of Residuals](image1)

![Residuals Q-Q plot](image2)

![Residuals boxplot](image3)

![Residuals plot](image4)
Intervals for the CPFH estimate

\[
\text{Prob} \left[ \hat{Y}_0 - s(\hat{Y}_0) t_{n-p, \frac{a}{2}} \leq Y \leq \hat{Y}_0 + s(\hat{Y}_0) t_{n-p, \frac{a}{2}} \right] = 1 - a
\]

where: 
\[
s^2(\hat{Y}_0) = STE^2 \left[ 1 + X_0'(X'X)^{-1}X_0 \right], \quad \text{for prediction interval}
\]
\[
s^2(\hat{Y}_0) = STE^2 \left[ X_0'(X'X)^{-1}X_0 \right], \quad \text{for confidence interval}
\]

and \( Y = \log(CPFH) \)
CPFH point estimate for an “unknown” system

F-35A empty weight = 29,098 lb
F135-PW-100 specific fuel consumption = 1.95 lb/lbf·h

\[
\log(CPFH) = \beta_0 + \beta_1 \cdot \log(29,098) + \beta_2 \cdot \log(1.95)
\]
CPFH estimate for the F-35A

**Expected CPFH:** 7,704 €

**95% CI:** 6,128 to 9,575 €
Comparison between “unknown” aircraft

<table>
<thead>
<tr>
<th>CPFH</th>
<th>JAS-39C</th>
<th>F-35A</th>
<th>Su-27SK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected</td>
<td>5,413</td>
<td>7,704</td>
<td>8,520</td>
</tr>
<tr>
<td>95% CI lower</td>
<td>4,413</td>
<td>6,128</td>
<td>6,714</td>
</tr>
<tr>
<td>95% CI upper</td>
<td>6,579</td>
<td>9,575</td>
<td>10,679</td>
</tr>
</tbody>
</table>
Comparison between “unknown” aircraft

\[ \text{Prob}(CPF_{F-35A} > CPF_{JAS-39C}) = 98.11\% \]
\[ \text{Prob}(CPF_{F-35A} - CPF_{JAS-39C} > 2,263 \, \text{€}) = 50\% \]

\[ \text{Prob}(CPF_{Su-27SK} > CPF_{F-35A}) = 71.03\% \]
\[ \text{Prob}(CPF_{Su-27SK} - CPF_{F-35A} > 801 \, \text{€}) = 50\% \]
## Review of the selected model

<table>
<thead>
<tr>
<th><strong>Constraints &amp; requirements</strong></th>
<th><strong>Results</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Use the sample of 22 aircraft operated by the Hellenic Air Force.</td>
<td>OK.</td>
</tr>
<tr>
<td>Use the appropriate cost information.</td>
<td><strong>OK.</strong> Current CPFH data used, excluding the <em>indirect support</em> cost category.</td>
</tr>
<tr>
<td>Use cost drivers (independent variables) that are easily accessible and quantifiable.</td>
<td><strong>OK.</strong> The cost drivers are physical and performance characteristics.</td>
</tr>
<tr>
<td>The model must be as less complex as possible and include no more than two cost drivers.</td>
<td><strong>OK.</strong> The selected model includes two independent variables.</td>
</tr>
<tr>
<td>The model should be statistically significant at the 5% level.</td>
<td><strong>OK.</strong> <em>p</em>-value = 3·10⁻⁸</td>
</tr>
<tr>
<td>The model should capture at least 75% of the CPFH variance.</td>
<td><strong>OK.</strong> <em>R²</em>ₐₐⱼ = 0.82</td>
</tr>
<tr>
<td>The model’s confidence and prediction intervals must be valid.</td>
<td><strong>OK.</strong> The residuals pass all tests</td>
</tr>
<tr>
<td>The model’s mathematical expression should make sense.</td>
<td><strong>OK.</strong> The model suggests that the aircraft weight and the engine specific fuel consumption correlate positively with the CPFH.</td>
</tr>
</tbody>
</table>
Post-Analysis considerations

• Small sample  →  high uncertainty, wide intervals
• Diverse systems  →  poor precision, robust CERs
• Residuals pass all tests  →  unbiased model, valid intervals
• Tailored model  →  no generalizations
  ❖ Why was the model built?
  ❖ Which question does the model actually answer?
  ❖ How does the model perform on the training sample?
  ❖ How can the model be useful?
REFERENCES