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THE ROYAL SOCIETY

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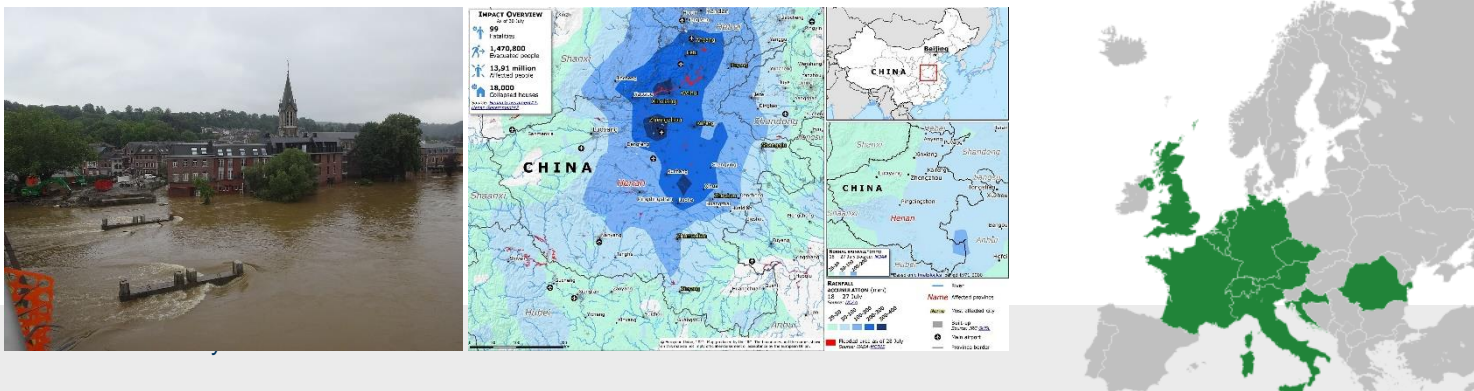
Multi-hazard risks of compound hydroclimatic extremes under uncertainty

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Background

- Extreme hydroclimatic events → led to severe losses (e.g. 2021 European Floods, Henan Floods)
- Many hydroclimatic extremes may be correlated → leading to compound extremes with multiple attributes
- The IPCC AR6 defined compound events may not necessarily result from dependent drivers

The impacts of concurrent extremes on land area have increased with a high confidence (IPCC AR6)



Background

- The compound extremes may trigger significant consequences much larger than the sum of impacts from individual extremes alone
- Without considering compound extremes, risk reduction efforts targeting one type of hazard may increase exposure and vulnerability to other hazards, in the present and future

Background

- Uncertainty exists in both univariate and multivariate hydrologic risk inference
- These uncertainties come from various sources
 - Data availability (e.g. 30, 50, 100-year records)
 - Model selection (e.g. Marginals, Copula)
 - Parameters
- Lead to imprecise risk inferences
 - Same flood with different RPs
 - Same RP with different flood magnitude

Address uncertainties in multi-hazard risk analyses

Background

- Consider one hydrological hazard has d correlated attributes (e.g. peak, volume), the joint probability is:

$$F(x_1, \dots, x_d | \gamma_1, \dots, \gamma_d, \theta) = C(F_1(x_1 | \gamma_1), \dots, F_d(x_d | \gamma_d) | \theta)$$

- Uncertain structure in marginal distributions F
- Uncertain structure in copula function C
- Parameter uncertainties in marginal (i.e. γ) and dependence (i.e. θ) structures

Name	Probability density function
GEV	$(\frac{1}{\sigma}) \exp(-(1 + k \frac{(x-\mu)}{\sigma})^{-\frac{1}{k}})(1 + k \frac{(x-\mu)}{\sigma})^{-1-\frac{1}{k}}$
Lognormal 1	$\frac{1}{x\sqrt{2\pi\sigma_y}} \exp(-\frac{(y-\mu_y)^2}{2\sigma_y^2})$ $y = \log(x)$, x greater than 0, $-\infty < \mu_y < \infty$, σ_y greater than 0
Pearson Type III	$\frac{1}{b^a\Gamma(a)}(x-\alpha)^{a-1}e^{-\frac{x-\alpha}{b}}$, $\Gamma(a) = \int_0^\infty u^{a-1}e^{-u}du$

Copula Name	Function [$C_\theta(u_1, u_2)$]
Clayton	$[u_1^{-\theta} + u_2^{-\theta} - 1]^{-1/\theta}$
Gumbel	$\exp \{ - [(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{1/\theta} \}$
Frank	$-\frac{1}{\theta} \ln \{ 1 + \frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{e^{-\theta}-1} \}$

Objective

Challenges:

1. Quantify parameter uncertainties in hydrologic risk models
2. Characterize contributions from different uncertain factors to imprecise risk inferences



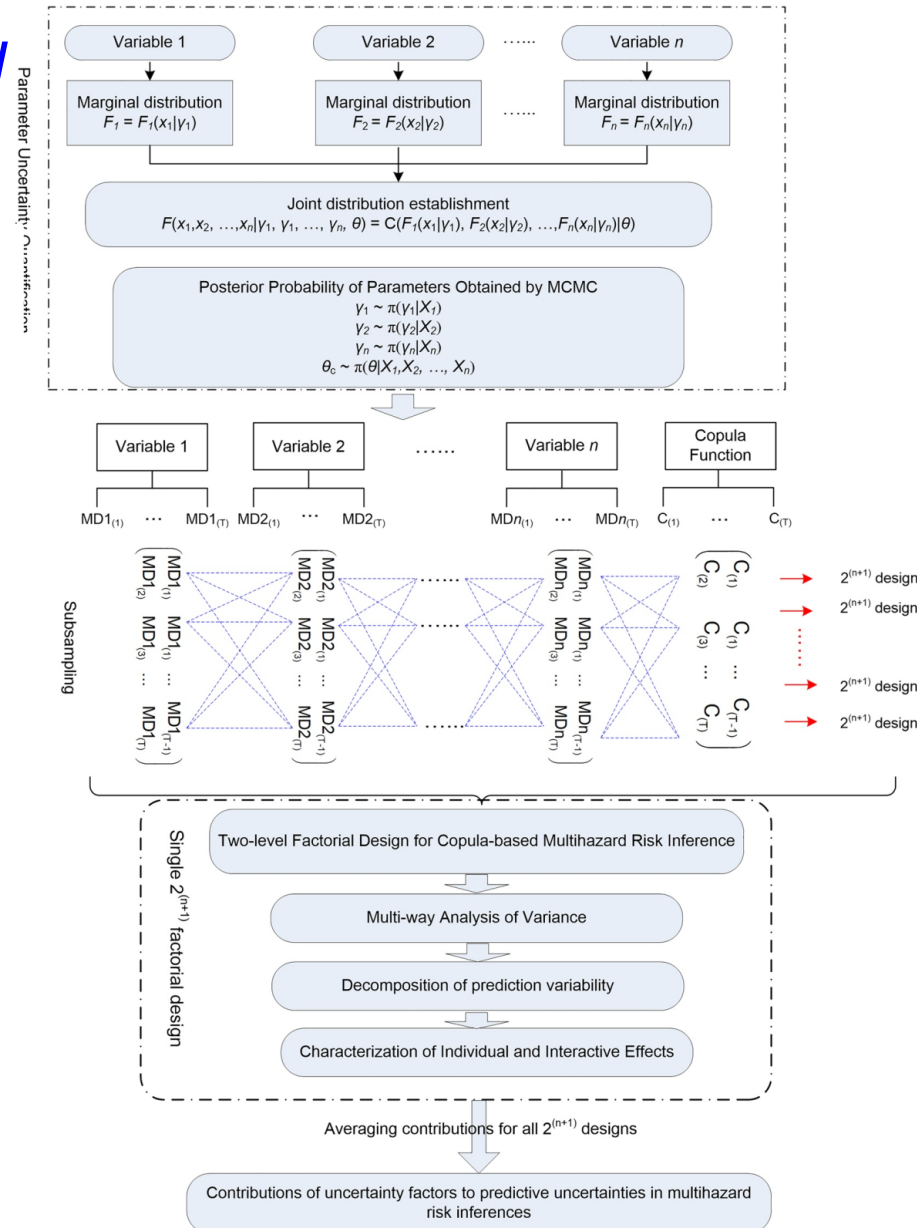
Iterative Factorial multimodel Bayesian copula (IFMBC)

- MCMC for parameter uncertainties in the copula model with different marginal and dependence structures
- Iterative factorial analysis for uncertainty partition from marginals, dependence structures and parameters

Methodology

an iterative factorial multimodel Bayesian copula framework was developed

- The **copula model** is applied for multivariate risk inference
- **AM-based MCMC** is employed to quantify parameter uncertainties in both marginal and dependent structures
- An **iterative factorial analysis** is developed to reveal the dominant contributors



Methodology-Copula model

- Consider one hydrological hazard has d correlated attributes (e.g. peak, volume), the joint probability is:

$$F(x_1, \dots, x_d \mid \gamma_1, \dots, \gamma_d, \theta) = C(F_1(x_1 \mid \gamma_1), \dots, F_d(x_d \mid \gamma_d) \mid \theta)$$

- Joint RP and the associate failure probability in “OR”, “AND”, and “Kendall”

$$T_{u_1, u_2}^{OR} = \frac{\mu}{1 - C_{U_1 U_2}(u_1, u_2 \mid \theta)}$$

$$p_T^{OR} = 1 - (C_{U_1 U_2}(u_1^*, u_2^* \mid \theta))^T$$

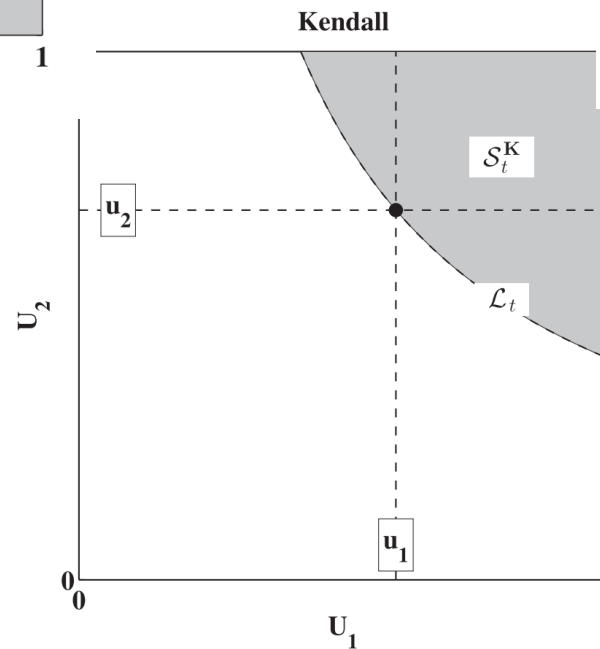
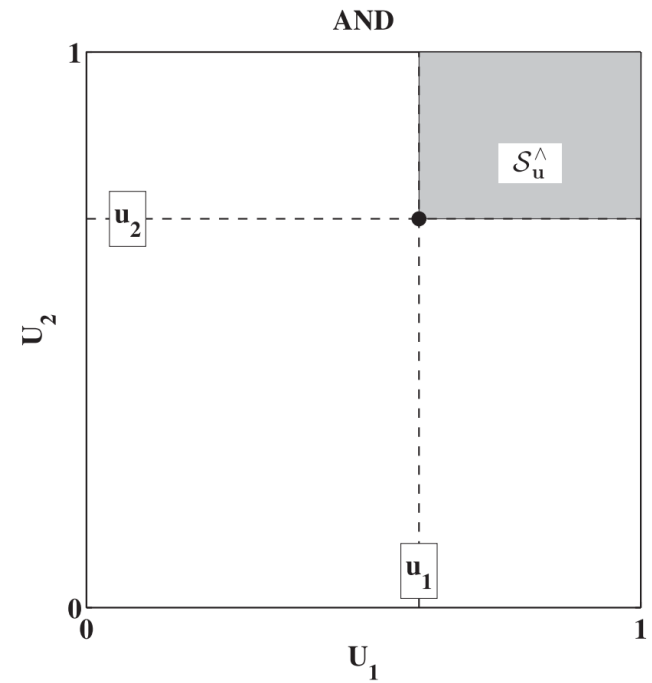
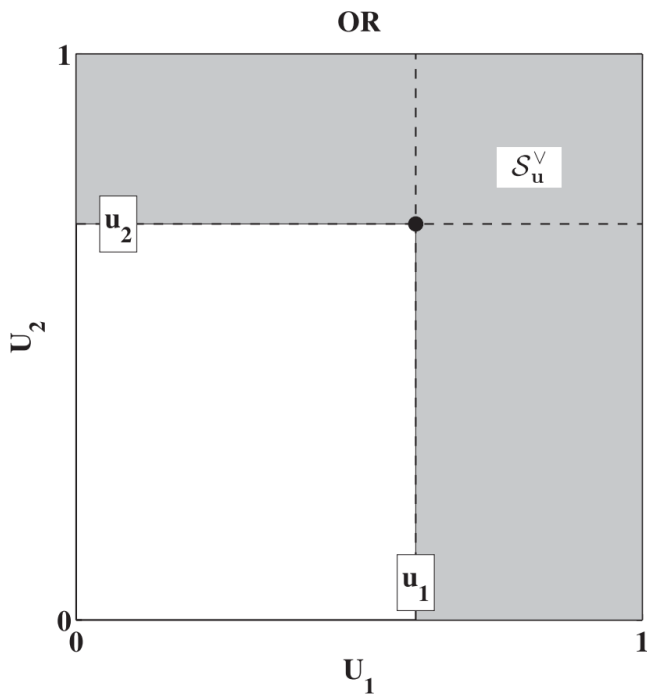
$$T_{u_1, u_2}^{AND} = \frac{\mu}{1 - u_1 - u_2 + C_{U_1 U_2}(u_1, u_2 \mid \theta)}$$

$$p_T^{AND} = 1 - (u_1^* + u_2^* - \hat{C}_{U_1 U_2}(u_1^*, u_2^* \mid \theta))^T$$

$$T_{u_1, u_2}^{Kendall} = \frac{\mu}{1 - P(C_{U_1 U_2}(u_1^*, u_2^*) \leq t)}$$

$$p_T^{Kendall} = 1 - (P(C_{U_1 U_2}(u_1^*, u_2^* \mid \theta) \leq t))^T$$

Methodology-Copula model



Methodology-IFA

- One critical process in IFA is to conduct a subsampling procedure for the uncertain factors to decompose the multiple levels for one factor into a number of two-level pairs
- P3, LN3, and LLOGIS3 (i.e., 3 levels in total) would be considered as the candidate marginals in this study, thus all possible 2-level pairs can be formulated as

$$: \begin{pmatrix} & P3 & P3 & LN3 \\ LN3 & LLOGIS3 & LLOGIS3 & \end{pmatrix}$$

Case Study

- Location:
Washington,
Philadelphia
- Extreme events:
River discharges
and Sea level

Table 1

Data Sources and Information for the Studied Cases

Location			
Variable		Washington, DC	Philadelphia, PA
River flow	River name	Potomac	Delaware
	USGS ID	1646500	1463500
	Duration	1931–2018	1913–2018
	Mean (m ³ /s)	3,480	2,500
	Skewness	2.05	1.66
	Kurtosis	7.78	6.98
	Range (m ³ /s)	[824, 12,063]	[875, 7,900]
Water level	Tide station	Washington	Philadelphia
	NOAA ID	8594900	8545530
	Duration	1931–2018	1913–2018
	Mean (m)	2.77	3.5
	Skewness	1.96	0.40
	Kurtosis	7.45	2.65
	Range (m)	[2.12, 4.76]	[2.84, 4.32]
	Vertical datum	Station datum	Station datum

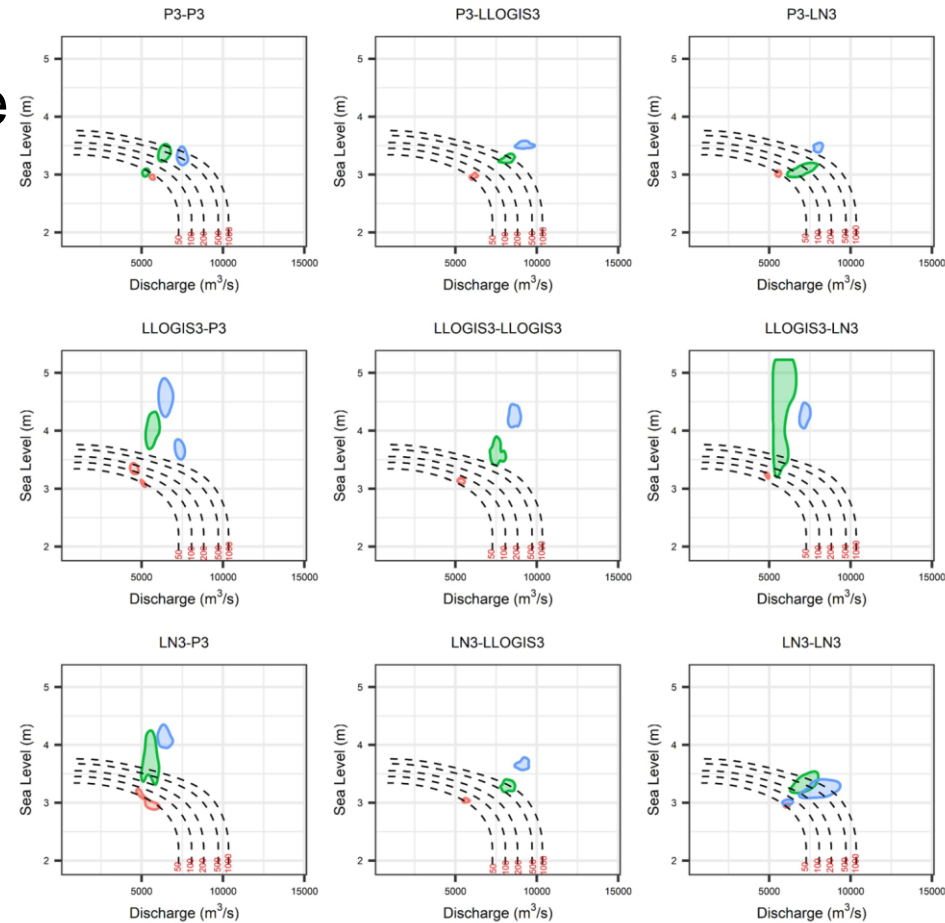
Case Study

- the compound extremes: annual maximum discharge the highest sea level within ± 1 day of the fluvial flood event.
- Marginal distributions for individual variables: P3, LLOGIS3, and LN3 distributions
- Copula model: Joe, Gumbel, Frank

Copula Name	Function [$C_\theta(u_1, u_2)$]	$\theta \in$	Generating functions [$\phi(t)$]
Joe	$1 - [(1 - u_1)^\theta + (1 - u_2)^\theta - (1 - u_1)^\theta (1 - u_2)^\theta]^{1/\theta}$	$[1, \infty)$	$-\ln(1 - (1 - t)^\theta)$
Gumbel	$\exp\{-[(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{1/\theta}\}$	$[1, \infty)$	$(-\ln t)^\theta$
Frank	$-\frac{1}{\theta} \ln\{1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1}\}$	$[-\infty, \infty) \setminus \{0\}$	$-\ln[\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}]$

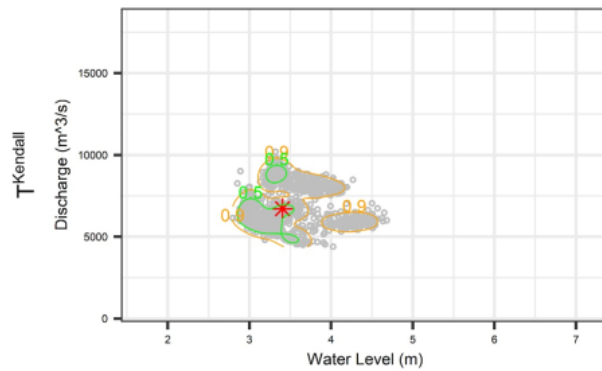
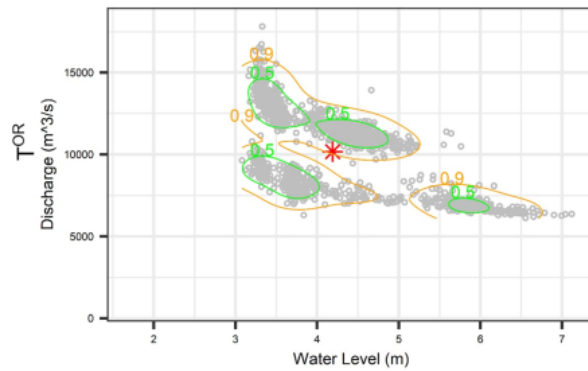
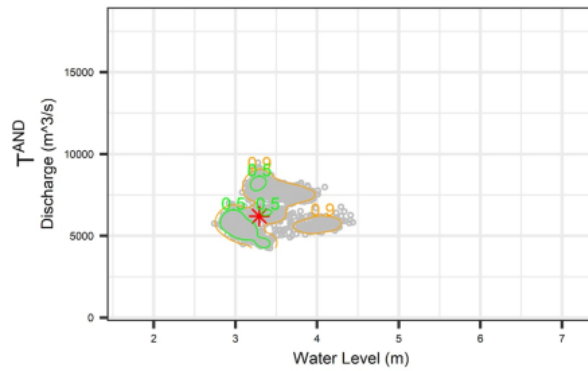
Results

- Uncertainties in multivariate risk inferences
- Different risk indices have different uncertainty features
- This may be due to diverse effects from one factor on different indices

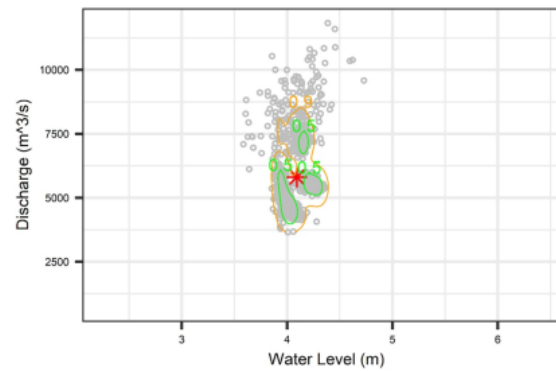
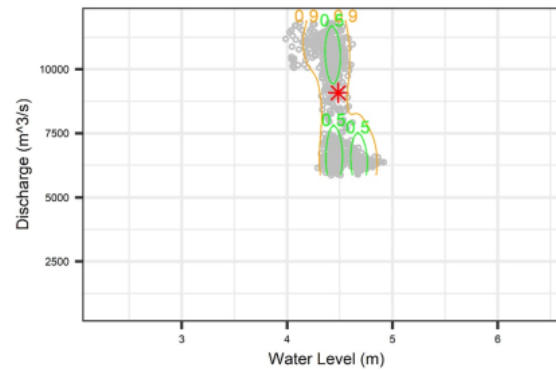
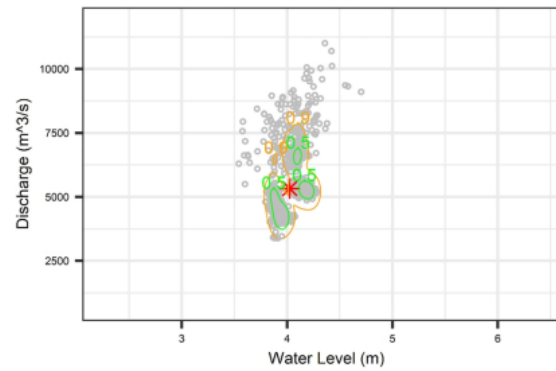


Results

(a) Washington



(b) Philadelphia



Results

- In this study, three uncertain factors were considered in IFA, including two marginal distributions and one copula function.
- each factor has three levels consisting of different marginal (i.e., P3, LLOGIS3, LN3) or dependence (i.e., Gumbel, Frank, and Joe copula) structures
- a total number of 27 two-level experimental designs in the IFA process

Models	Marginal for flood	Marginal for seal level	Dependence structure
1	P3	P3	Gumbel
2	P3	P3	Frank
3	P3	LN3	Gumbel
4	P3	LN3	Frank
5	LN3	P3	Gumbel
6	LN3	P3	Frank
7	LN3	LN3	Gumbel
8	LN3	LN3	Frank

- The copula function dominates the predictive uncertainties in failure probability in AND, may be higher than 25%
- Parameter uncertainties is the second significant contributor

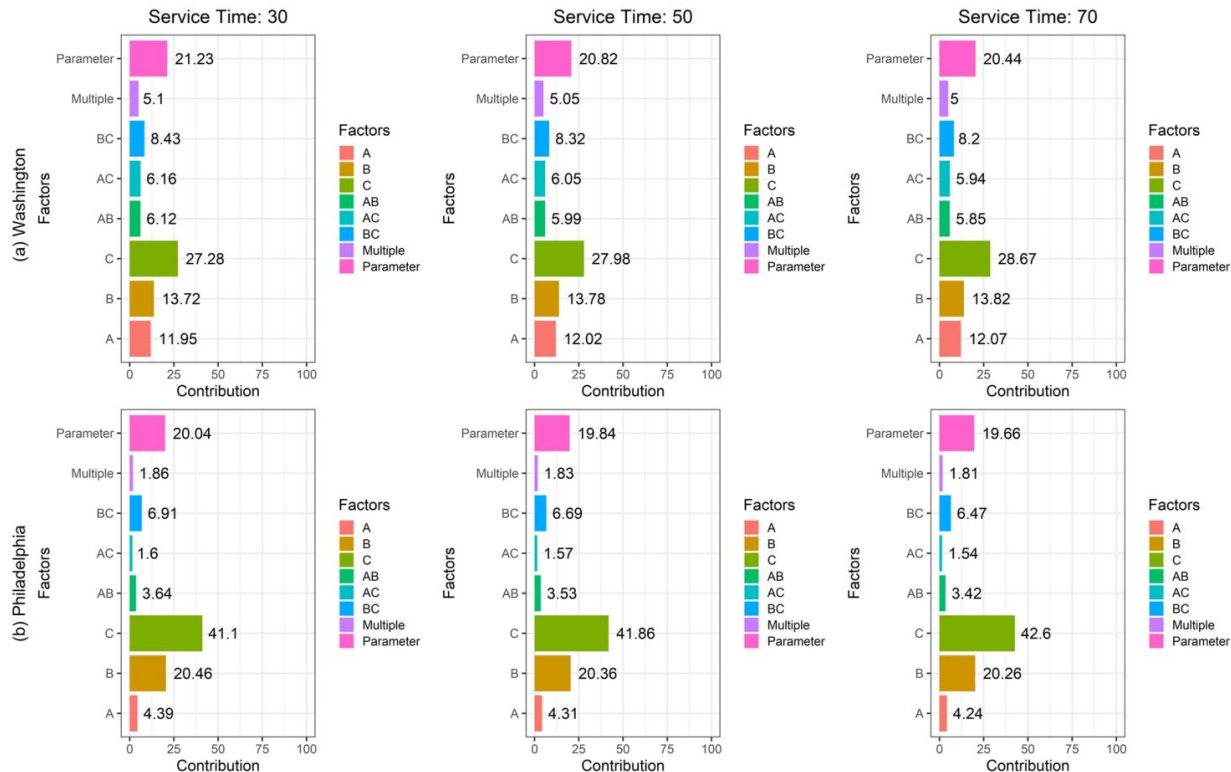


Table 3

The First Two Contributors for the Predictive Uncertainties in the Multi-Hazard Risk Inferences for Different Failure Probabilities With a 30 yr Service Time

Stations		Washington	Philadelphia
FP in AND	1st	Copula	Copula
	2nd	Parameter	Marginal for discharge
FP in OR	1st	Parameter	Parameter
	2nd	Marginal for sea level	Marginal for discharge
FP in Kendall	1st	Copula	Parameter
	2nd	Parameter	Copula



Fan Y.R.*, Yu L., Shi X. et al., (2021). Tracing uncertainty contributors in the multi-hazard risk analysis for compound extremes. *Earth's Future*, 9, e2021EF002280

Thank you!

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